PINNER WOOD SCHOOL

MATH CALCULATION POLICY

Approval Authority	
Effective From:	1st September 2021
Date Ratified by GB:	
Next Review Date:	1st September 2022
Signed by Chair of GB:	

Pinner Wood Maths Written Calculation Policy 2020-2021

This policy supports the White Rose maths used throughout the school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This calculation policy should be used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

- Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.
- Pictorial representation – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations.

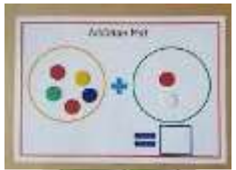
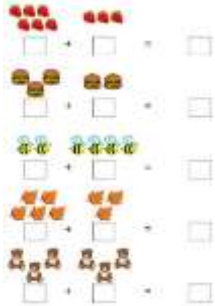
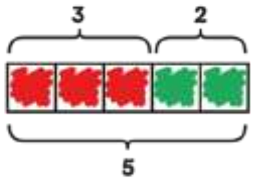
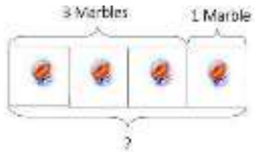
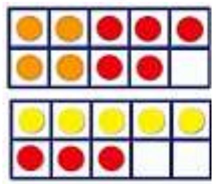
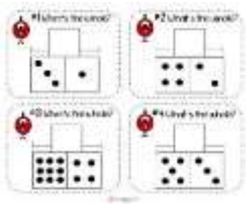
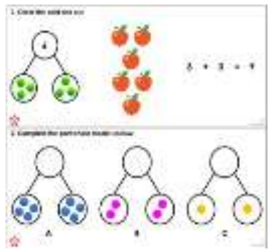
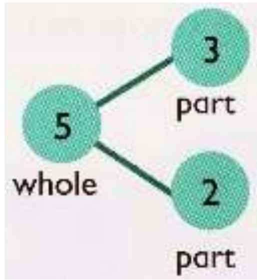
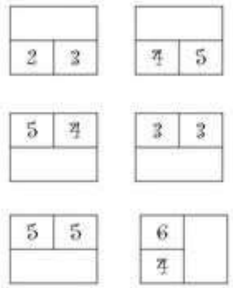
Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

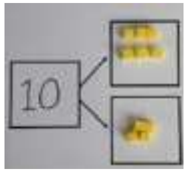
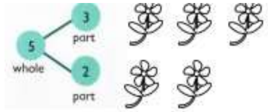
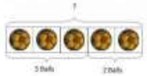
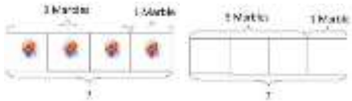
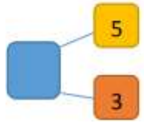
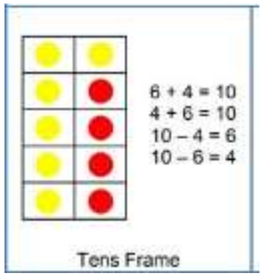
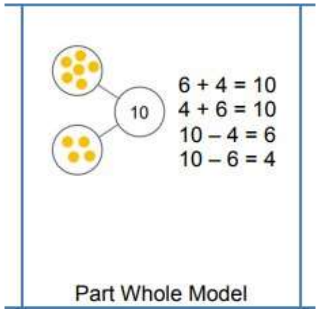
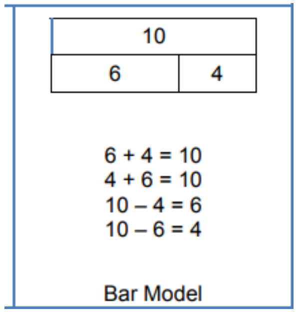
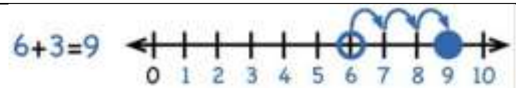
How to Use the policy:

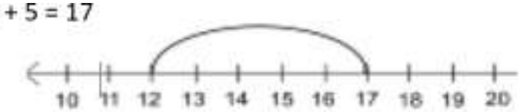
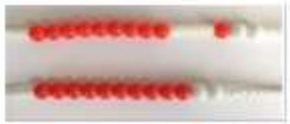
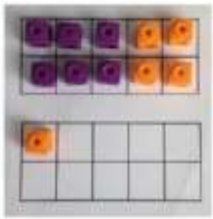
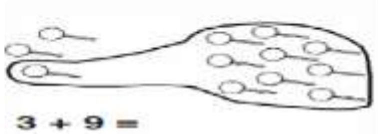
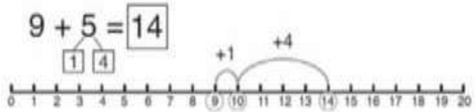
This mathematics policy is a guide for all staff at Pinner Wood and has been adapted from work by the NCETM. All teachers have access to White Rose Maths and we use Collins Maths to do the teaching from. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used.
For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

Addition

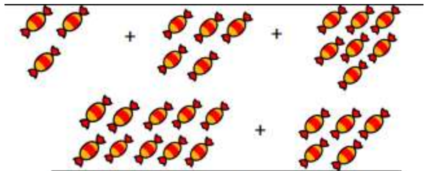
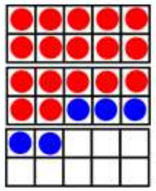
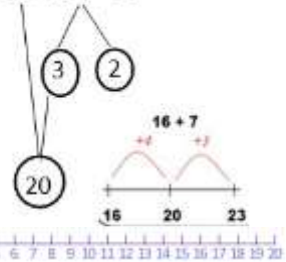
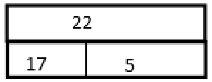
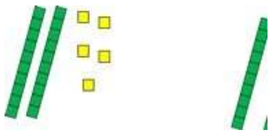
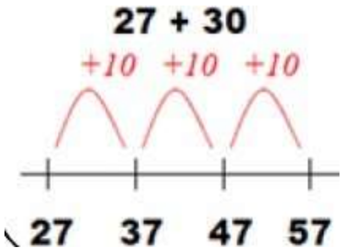
Addition- EYFS			
Objectives	**Concrete**	**Pictorial**	**Abstract**
- Knows that a group of things change in quantity when something is added. - Find the total number of items in two groups by counting all of them. - Says the number that is one more than a given number. - Finds one more from a group of up to five objects, then ten objects. - In practical activities and discussion, beginning to use the vocabulary involved in adding. - Using quantities and objects, they add two single digit numbers and count on to find the answer. - Solve problems including doubling.	   Use toys and general classroom resources for children to physically manipulate, group/regroup    Use specific maths resources such as counters, snap cubes, Numicon etc.    Use visual supports such as ten frames, part part whole and addition mats, with the physical objects and resources that can be manipulated.	  Two groups of pictures so children are able to count the total.   Bar model using visuals, pictures/icons or colours.  Use visual supports such as ten frames, part part whole and addition mats with pictures/icons.  	A focus on symbols and numbers to form a calculation. $5+2=7$   * No expectation for children to be able to record a number sentence/addition calculation.

Addition- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 Use cubes to add two numbers together as a group or in a bar. (Some children may still need to use real objects)  Use part-part whole model	 Use pictures to add two numbers together as a group or in a bar.   <u>The Bar Model</u> will be continued from EYFS as a method to support problem solving involving addition, continuing with the concrete representations and moving onto using pictorial representations of objects. Some children will also move onto the abstract.  Pictorial (concrete) Abstract	 Use the part-part whole diagram as shown above to move into the abstract. **4 + 3 = 7** **10= 6 + 4**
Represent and use number bonds and related subtraction facts within 20	 Tens Frame (Some children may need to initially use real objects then move onto the representation, egg boxes may also be used to support this)	 Part Whole Model	 Bar Model Bar model and part-part whole to be used alongside abstract
Addition and subtraction of one-digit and two-digit numbers to 20 including 0.		 Start at the larger number on the number line and count on in ones.	5 + 12 = 17 17= 12 + 5

Start at the bigger number and counting on	Start with the larger number on the bead string and then count of to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10 (The 'Make 10' strategy)	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or Partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Vocabulary	add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole		

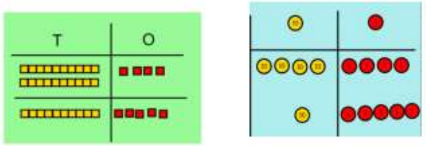
Addition- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
Adding 3 1-digit numbers	**4 + 7 + 6= 17** Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.
Adding a 2-digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32 *Use of bead strings/Dienes	17 + 5 = 22 Use part part whole and number line to model.  Number lines Bar Model 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Adding a 2-digit number and multiples of 10	 25 + 10 = 35 Explore that the ones digit does not change	 Base 10 may be used above the number line initially. The calculation will be shown alongside the number line to see the connection	27 + 10 = 37 27 + 20 = 47 27 + □ = 57

<table>
<tr>
<td>Adding two 2-digit numbers
(No re-grouping)</td>
<td>

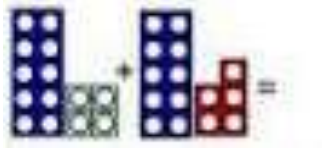
24 + 15=

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



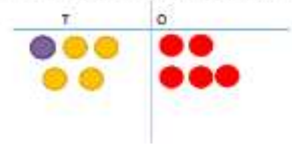
(Some children may not be ready for place value counters in Y2)

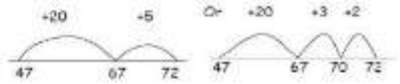
Numicon may also be used



</td>
<td>

After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.





Use number line and bridge ten using part whole if necessary.

Base 10 may be used above the number line.

The calculation will be shown alongside the number line to see the connection

<table>
<tr><th>Model</th><th>Calculation</th></tr>
<tr><td></td><td></td></tr>
</table>

</td>
<td>

Partitioning:

25 + 47
20 + 5   40 + 7
20 + 40 = 60
5+ 7 = 12
60 + 12   72

Recording addition in columns supports place value and prepares for formal written methods with larger numbers.

Toward the end of the year, children move to more formal recording using partitioning method:

40 + 7
30 + 5
70 + 12

Or

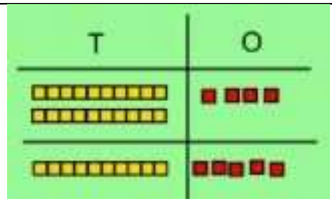
47 + 25

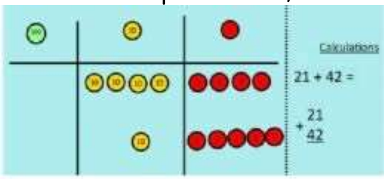
47 + 20 = 67
67 + 5 = 72

</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary</td>
</tr>
</table>

Addition- Year 3

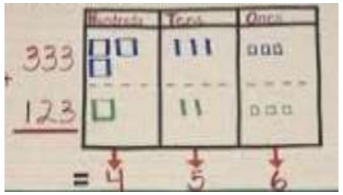
<table>
<tr><th>Objective and Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Add and subtract numbers with up to 3-digits, using formal written methods of columnar addition

Column addition (no regrouping)</td>
<td>

Using manipulatives (dienes, numicon, counters), children are to line up hundreds, tens and ones.


Children should be secure with using PV counters before moving onto pictorial.

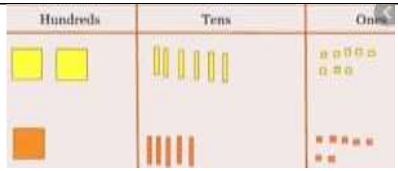
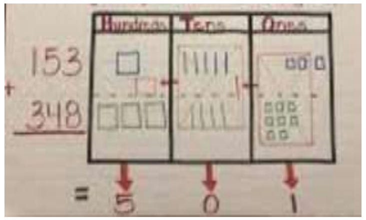
The calculation will be shown alongside the model used to see the connection

<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>

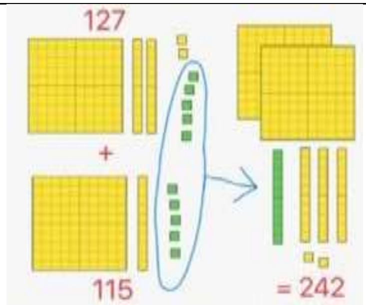
Children are to draw, in a PV frame, the manipulatives, that they are using.

Secure knowledge of representation with the PV columns.

The calculation will be shown alongside the model to see the connection

<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>$$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$$
Children to move onto recording more formally.
Some children may need to use the expanded method (see below).</td>
</tr>
<tr>
<td>Column addition (with regrouping)</td>
<td>
</td>
<td>

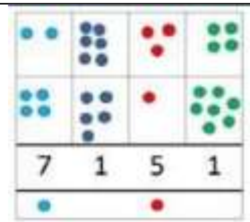
Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.</td>
<td>$$\begin{array}{rcl} 20 & + & 5 \\ 40 & + & 8 \\ \hline 60 & + & 13 \quad = 73 \end{array}$$
Children are to begin with the abstract: expanded form.
For those children, that are confident after AFL, the below method should be used.</td>
</tr>
</table>

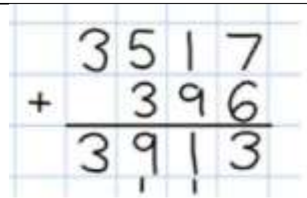
	 Exchange ten ones for a ten. Model using Dienes, Numicon and place value counters.		536 + 85 621 1 1
Vocabulary	addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary		

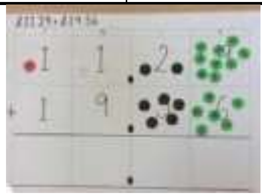
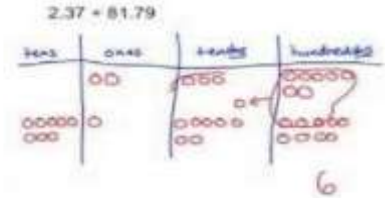
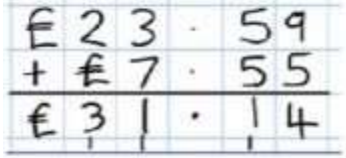
<table>
<tr><th colspan="4">Addition- Year 4</th></tr>
<tr><th>Objective and Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Using formal written methods of columnar addition where appropriate

add numbers with up to 4 digits (with exchange)</td>
<td>Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.

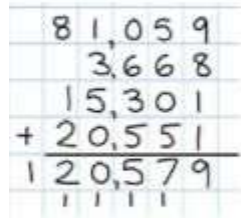
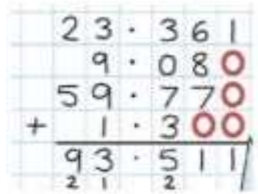

The calculation will be shown alongside the manipulative used to see the connection

<table><tr><th>Model</th><th>Calculatio
n</th></tr><tr><td></td><td></td></tr></table></td>
<td>

Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.</td>
<td>

Continue from previous work to carry hundreds as well as tens.</td>
</tr>
<tr>
<td>Add decimals with 2 decimal places, including money.</td>
<td>
Introduce decimal place value counters and model exchange for addition.</td>
<td>
</td>
<td>

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point</td>
</tr>
</table>

Addition- Year 5/6			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
add numbers with more than 4 digits.	See Year 4	See Year 4	 Children should have abstract supported by a pictorial or concrete if needed.
add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	See Year 4	See Year 4	  Insert zeros for place holders.
Vocabulary	addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point		

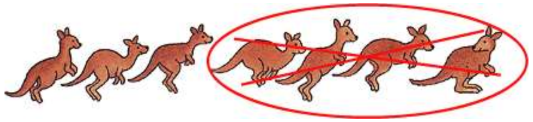
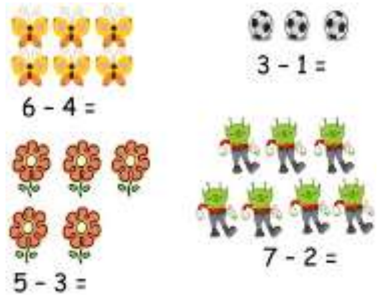
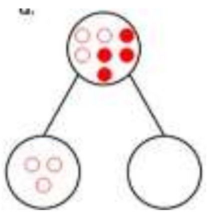
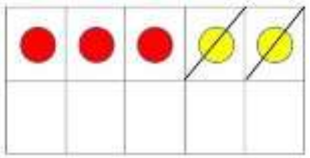
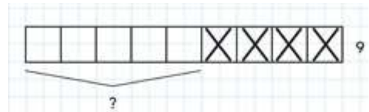
Subtraction

<table>
<tr><th colspan="4">Subtraction- EYFS</th></tr>
<tr><th>Objectives</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>- Knows that a group of things change in quantity when something is taken away

- Find one less from a group of five objects, then ten objects.

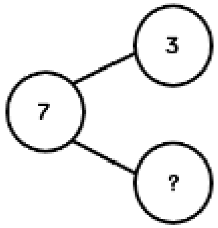
- In practical activities and discussion, beginning to use the vocabulary involved in subtracting.

- Using quantities and objects, they subtract two single digit numbers and count back to find the answer.</td>
<td>  
Use toys and general classroom resources for children to physically manipulate, group/regroup.

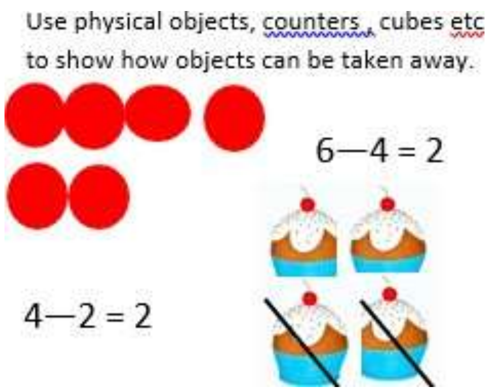
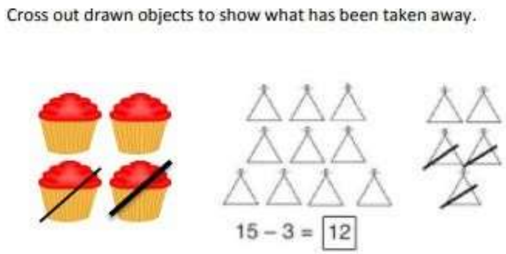
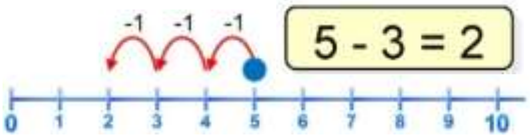
 
Use specific maths resources such as snap cubes,
Numicon, bead strings etc.
 
Use visual supports such as ten frames, part part
whole and subtraction mats, with the physical objects and resources that can be manipulated.</td>
<td>

A group of pictures for children to cross out or cover quantities to support subtraction.
 

Use visual supports such as ten frames, part part whole and bar model with pictures/icons.</td>
<td>A focus on symbols and numbers to form a calculation.

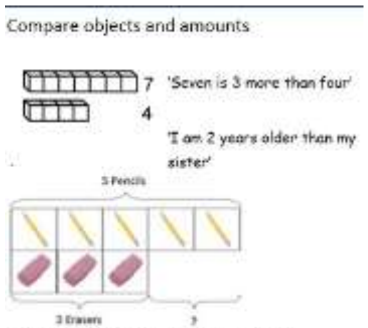
10 - 6 = 4

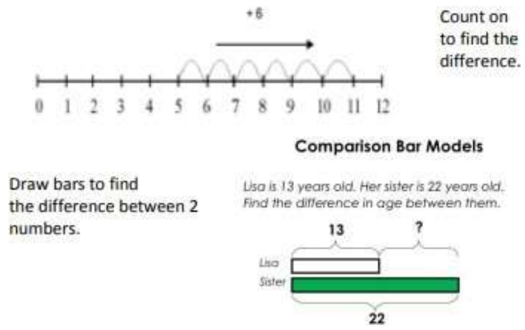
3 | ?
7

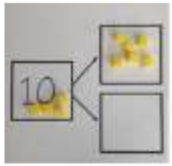
7 - 3 = ? 
* No expectation for children to be able to record a number sentence/addition calculation.</td>
</tr>
</table>

Subtraction- Year 1

Objective and Strategy	Concrete	Pictorial	Abstract
Subtract one-digit and two-digit numbers to 20, including 0. Taking away ones			7—4 = 3 16—9 = 7
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4**  Use counters and move them away from the group as you take ther away counting backwards as you go. 	 Count back on a number line or track Start at the bigger number and count back the smaller number showing the jumps on the number line.	Put 13 in your head, count back 4. What number are you at? (Use your fingers to help you)

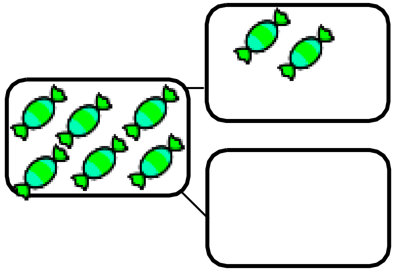
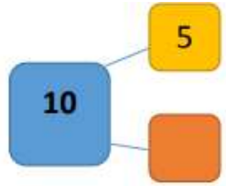
<table>
<tr>
<td>Find the difference</td>
<td>

Lay objects to represent bar model.</td>
<td>
</td>
<td>Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister?</td>
</tr>
<tr>
<td>Represent and use number bonds and related subtraction facts within 20

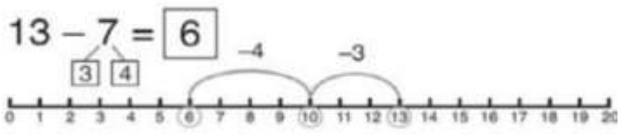
Part-part whole model</td>
<td>

Link to addition. Use PPW model to model the inverse.

If 10 is the whole and 6 is one of the arts, what s the other part?

10—6 = 4</td>
<td>
Use a pictorial representation of objects to show the part-part whole model</td>
<td>

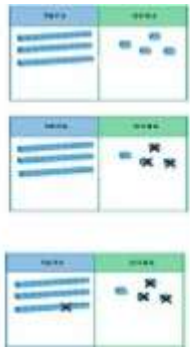
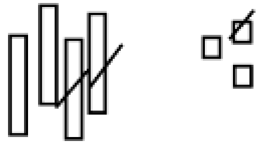
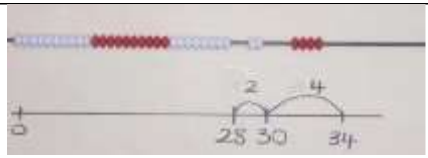
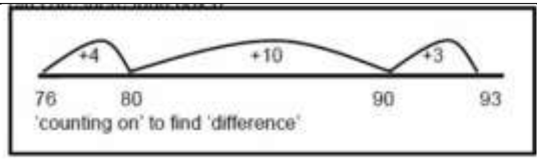
Move to using numbers within the part whole model.</td>
</tr>
<tr>
<td>Make 10</td>
<td>14 – 9 =

Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</td>
<td>

Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</td>
<td>16 – 8=

How many do we take off to reach the next 10?
How many do we have left to take off?</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…</td>
</tr>
</table>

Subtraction- Year 2

Objective and Strategy	Concrete	Pictorial	Abstract
Subtract a two-digit number and ones, a two-digit number and tens, two two-digit numbers Partitioning to subtract without re-Grouping: *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping. The calculation will be shown alongside the manipulative used <table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table>	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22 *Recording subtraction in columns supports place value and prepares for formal written methods with larger numbers.* Toward the end of the year, children move to more formal recording using partitioning method: e.g. 43-21=22 43 – 21 = 43 – 20 = 23 23 – 1 = 22
Make ten strategy	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units		

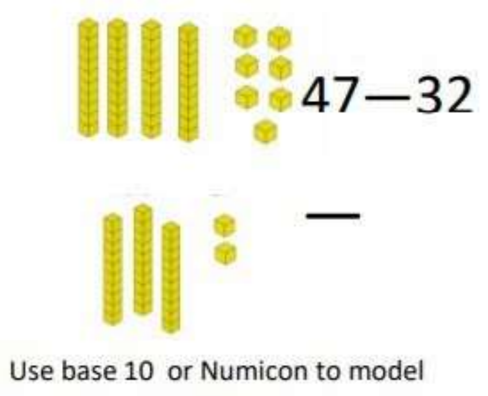
Subtraction- Year 3

Objective and Strategy

To subtract numbers with up to three-digits, using formal written methods of columnar subtraction

Column subtraction (without exchanging)

Concrete

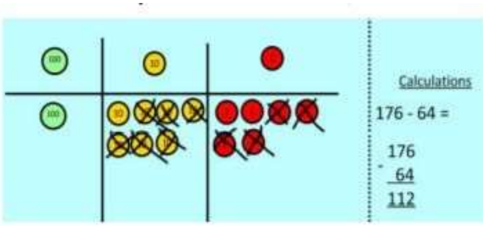


The calculation will be shown alongside the model chosen to see the connection

Model	Calculation

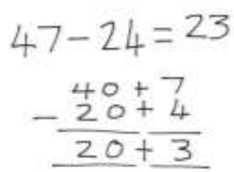
Pictorial

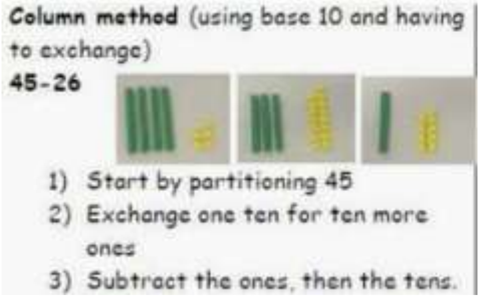
Children are to be secure with use of PV counters before moving onto abstract.

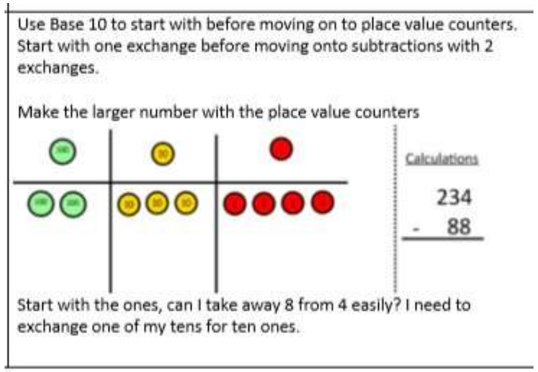


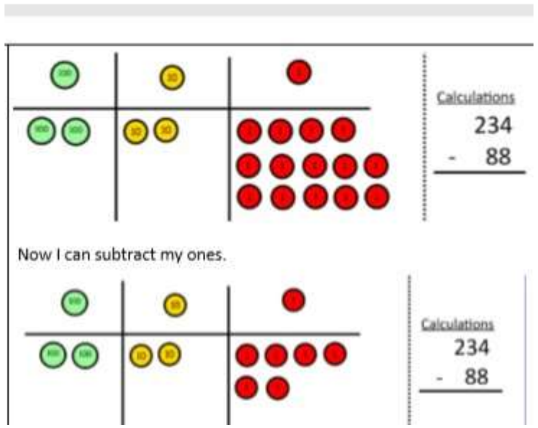
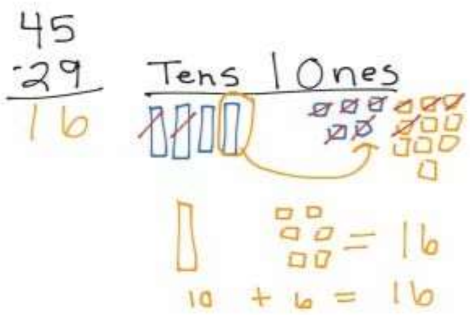
Abstract

Children should begin with the expanded form. Moving onto a more formal way as below.

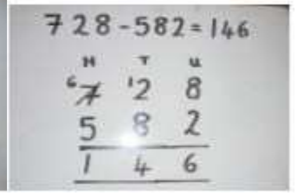


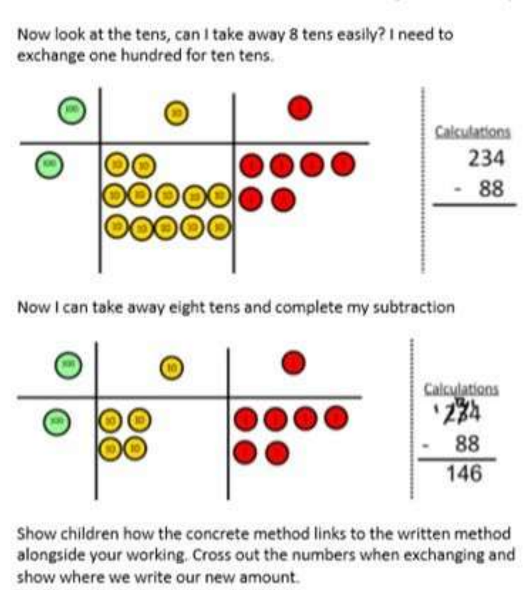
<table>
<tr>
<td>Column Subtraction (with exchanging)</td>
<td>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase ‘take and make’ for exchange.




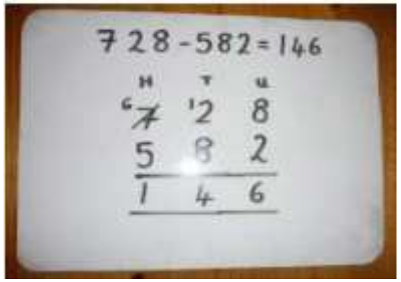

</td>
<td>

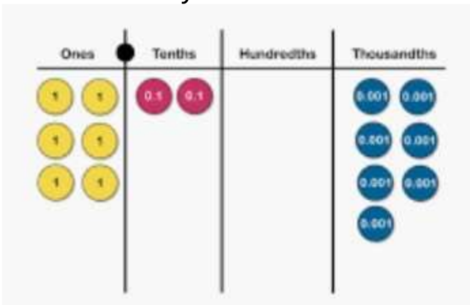
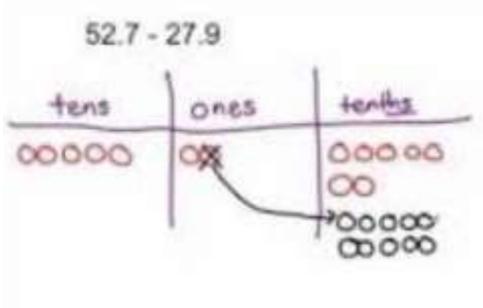
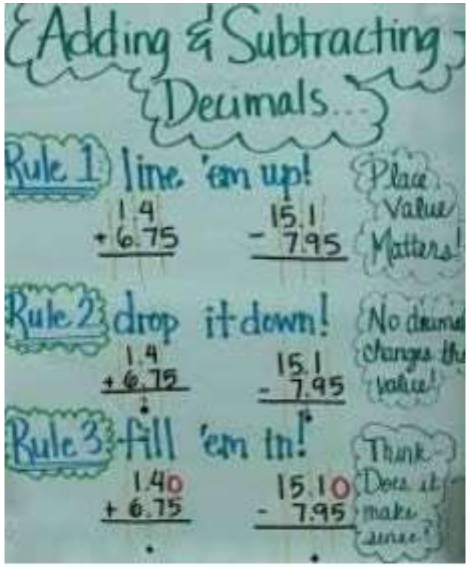
When confident, children can find their own way to record the exchange/regrouping</td>
<td>Children should begin with the expanded form. Moving onto a more foraml way as below (bottom picture).

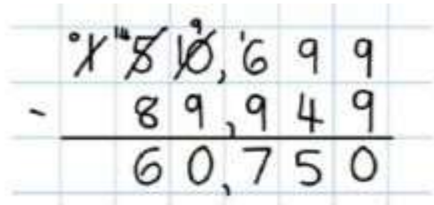
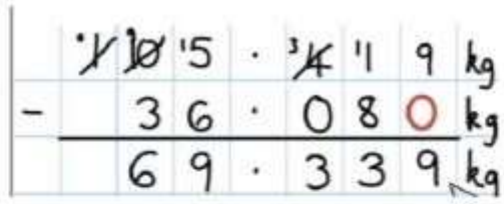


</td>
</tr>
</table>

	
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units

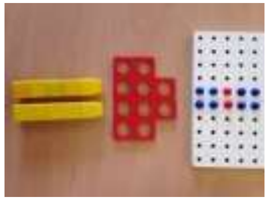
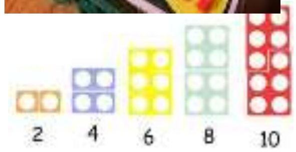
Subtraction- Year 4

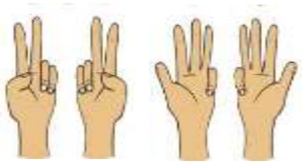
Objective and Strategy	Concrete	Pictorial	Abstract
Subtract numbers with up to 4 digits using the formal written methods appropriate of columnar subtraction where appropriate Year 4 subtraction with up to 4 digits.	Model process of exchange using Numicon, base ten and then move to PV counters. Use the phrase 'take and make' for exchange- see Y3 The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	Children to draw pv counters and show their exchange—see Y3 The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	 This will lead to an understanding of subtracting any number including decimals.

Introduce decimal subtraction through context of money	Children to be encourgaed to use counters to represent numbers and take counters away to subtract.  	 When confident, children can find their own way to record the exchange/regrouping	
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units		

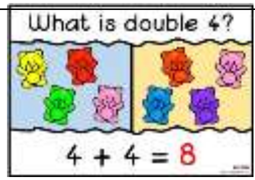
Subtraction- Year 5/6			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Subtract with at least 4 digits, including money and measures. Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	See Year 4	See Year 4	 
Vocabulary	equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units		

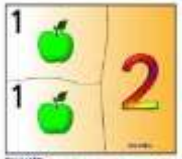
Multiplication

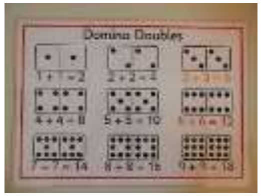
<table>
<tr><th colspan="4">Multiplication-
EYFS</th></tr>
<tr><th>Objectives</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>- Solve problems including doubling</td>
<td>



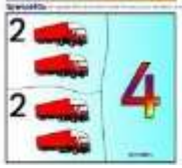
Counting and other maths resources for children to make 2 equal groups.




Physical and real life examples that encourage children to see concept of doubling as adding two equal groups.</td>
<td>

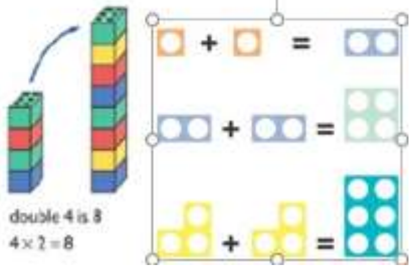
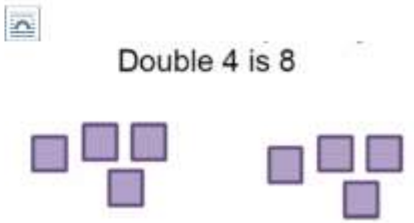
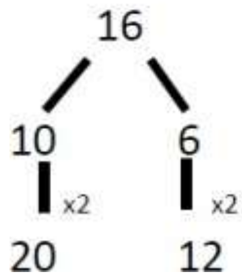
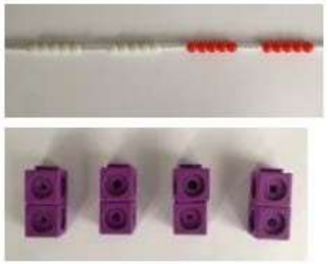
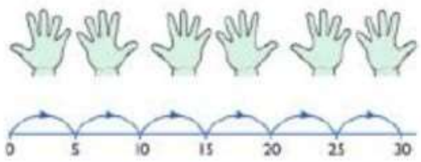
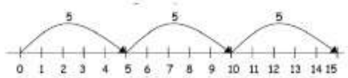


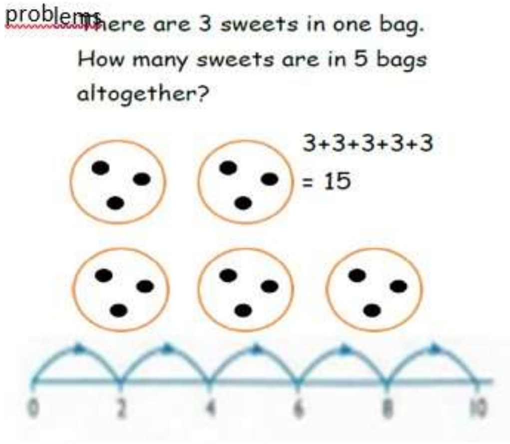
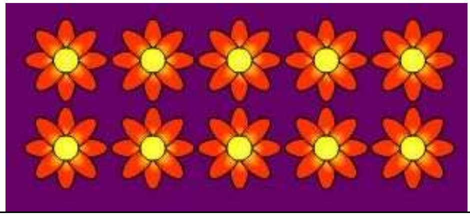


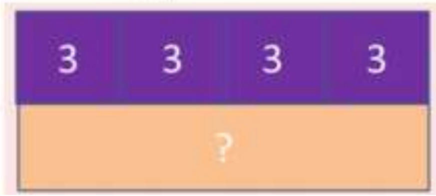
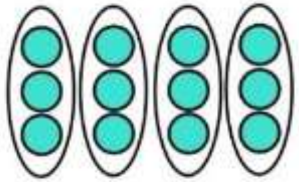
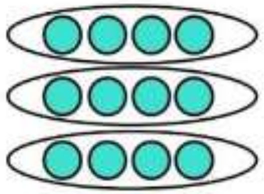


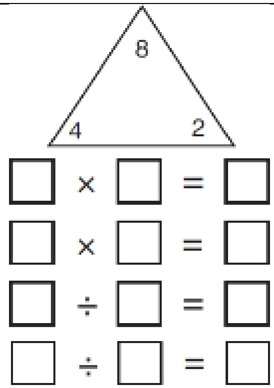
Pictures and icons that encourage children to see concept of doubling as adding two equal groups.</td>
<td><table>
<tr><td>1+1=</td><td>7+7=</td></tr>
<tr><td>2+2=</td><td>8+8=</td></tr>
<tr><td>3+3=</td><td>9+9=</td></tr>
<tr><td>4+4=</td><td>10+10=</td></tr>
<tr><td>5+5=</td><td>11+11=</td></tr>
<tr><td>6+6=</td><td>12+12=</td></tr>
</table>
Addition calculations to model adding two equal groups.</td>
</tr>
</table>

Multiplication- Year 1

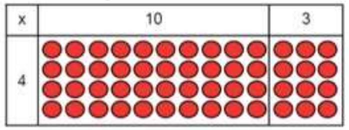
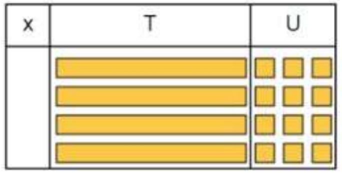
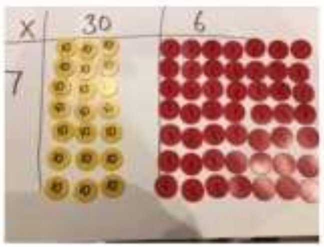
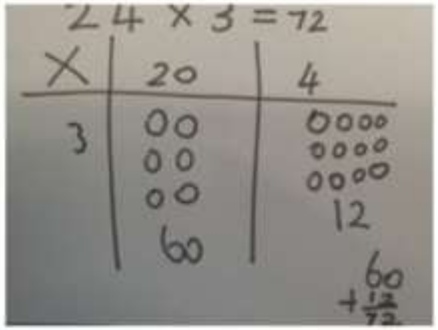
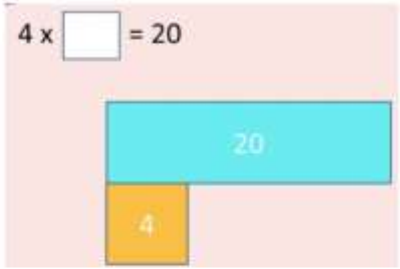
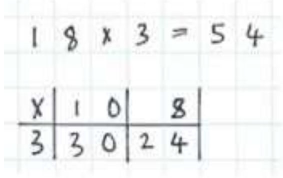
Objective and Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling  	Draw pictures to show how to double numbers  	 Partition a number and then double each part before recombining it back together.
Counting in multiples	 Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. **2, 4, 6, 8, 10** **5, 10, 15, 20, 25, 30**
Repeated addition	 Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?   5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

		Use pictorial including number lines to solve problems 	
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 		3 x 2 = 6 2 x 5 = 10
Vocabulary	Groups of, lots of, times, array, altogether, multiply		

Multiplication- Year 2			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.     	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*	 Sharing practically Fractions of shapes etc	 	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.
Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.		

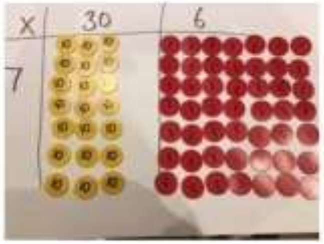
Multiplication- Year 3

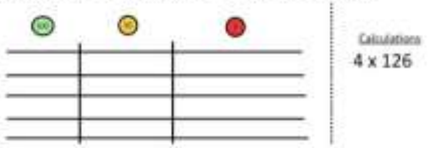
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Multiplying two digit number by a one digit number Grid method progressing to the formal method. Solving problems including missing number problems, integer scaling problems.	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.  Add up each column, starting with the ones making any exchanges needed. The calculation will be shown alongside the model chosen to see the connection **Model** | **Calculation**	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. TO x O  Children to add up each column to find the answer.
Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up		

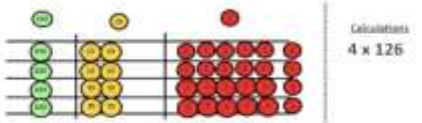
Multiplication- Year 4			
Objective and Strategy	Concret e	Pictori al	Abstra ct

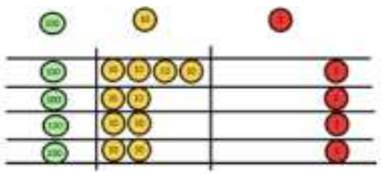
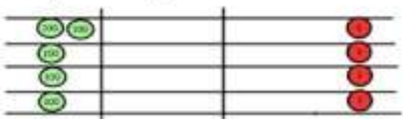
<table>
<tr>
<td>Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

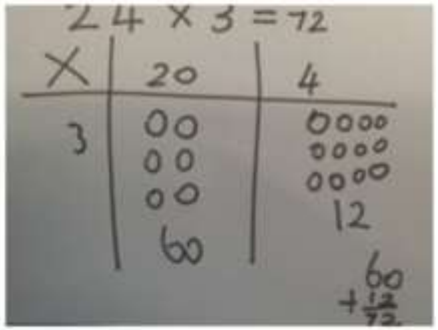
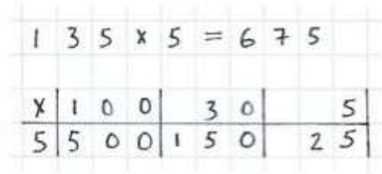
Grid method recap from year 3 for 2 digits x 1 digit

Multiplying numbers by 1 digit (year 4 expectation)</td>
<td>

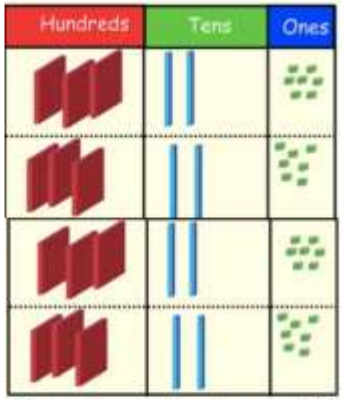
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.


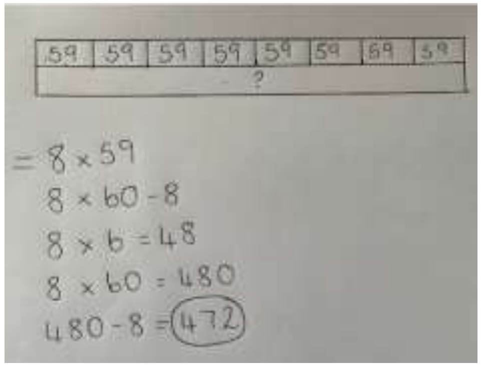
Fill each row with 126.


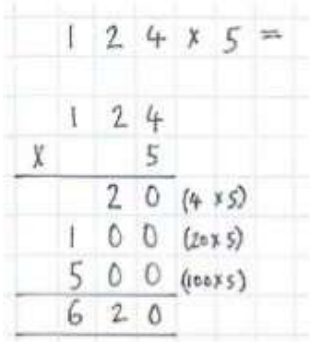
Add up each column, starting with the ones making any exchanges needed.

Then you have your answer.
</td>
<td>Children can represent their work with place value counters in a way that they understand.

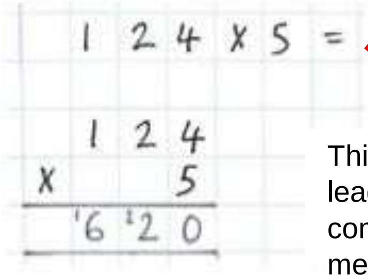
They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.

</td>
<td>HTO x O


Children to add up each column to find the answer.</td>
</tr>
</table>

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642


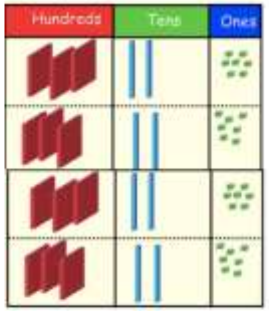
It is important at this stage that they always multiply the ones first.
The corresponding long multiplication is modelled alongside
<table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table></td>
<td>This grid method may be used to show how this relates to a formal written method.
<table><tr><th>x</th><th>100</th><th>20</th><th>4</th></tr><tr><td>5</td><td>500</td><td>100</td><td>20</td></tr></table>


Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</td>
<td>



This may lead to a compact method.</td>
</tr>
<tr>
<td>Vocabulary</td>
<td colspan="3">Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive</td>
</tr>
</table>

Multiplication Year 5

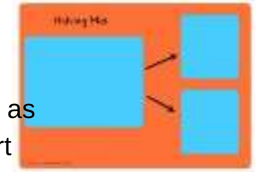
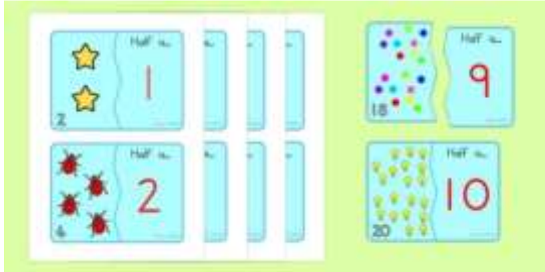
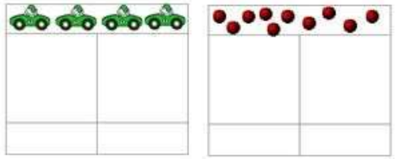
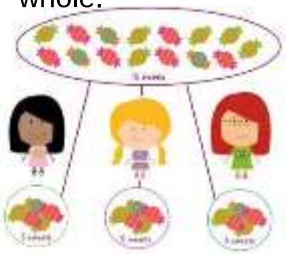
Objective and Strategy	Concrete	Pictorial	Abstract
Multiply numbers up to 4-digits by a one-digit number using the format written method, including long multiplication for 2-digit numbers Column multiplication for 3 and 4 digits x 1 digit	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>	327 x 4 28 80 1200 1308 3 2 7 x 4 1 3 0 8 1 2 This may lead to a compact method.
Column multiplication (long multiplication)	Manipulatives may still be used with the corresponding long multiplication modelled alongside <table><tr><td>**Model**</td><td>**Calculation**</td></tr><tr><td></td><td></td></tr></table>	Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 24 x 16 = 384 <table><tr><td>x</td><td>20</td><td>4</td></tr><tr><td>10</td><td>200</td><td>40</td></tr><tr><td>6</td><td>120</td><td>24</td></tr></table>	2 4 x 1 6 1 4 4 2 4 0 3 8 4 24 x 6 on the first row. (6 x 4 = 24, carrying the 2 for the 20, then 6 x 2) 24 x 10 on the second row. Show multiplying by 10 by putting zero in the units first. 1 2 3 4 x 1 6 7 4 0 4 (1234 x 6) 1 2 3 4 0 (1234 x 10) 1 9,7 4 4

Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed

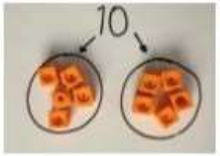
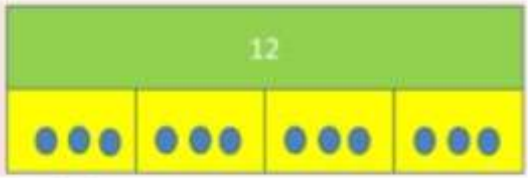
Multiplication- Year 6			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Multiply decimal up to 2 decimal place by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 3 · 1 9 × 8 2 5 · 5 2 1 7
vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed		

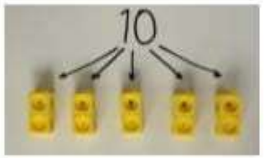
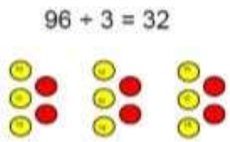
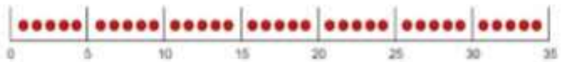
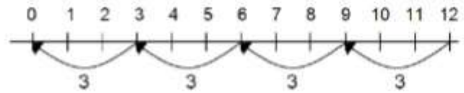
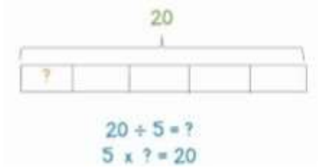
Division

Division- EYFS

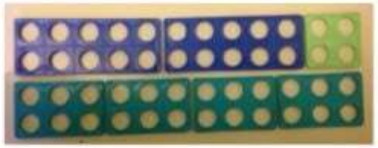
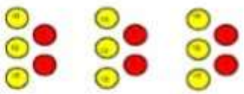
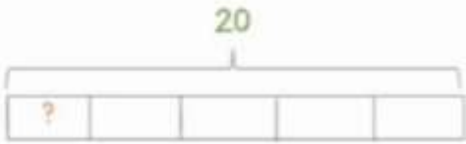
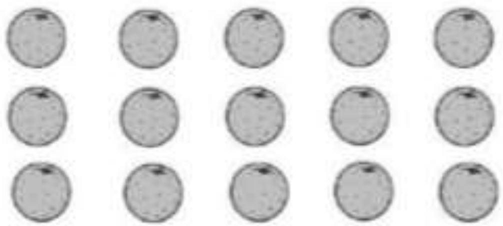
Objectives	Concrete	Pictorial	Abstract
Solve problems including halving and sharing. • Halving a whole, halving a quantity of objects. • Sharing a quantity of objects.	  Children have the opportunity to physically cut objects, food or shapes in half.  Counting and other maths resources for children to share into two equal groups.   Use visual supports such as halving mats and part part whole, with the physical objects and resources that can be manipulated.  Counting and other maths resources for children to explore sharing between 3 or more.	 Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.  Bar model with pictures or icons to support understanding of finding 2 equal parts of a number, to further understand how two halves make a whole.  Pictures for children to create and visualise 3 or more equal groups.	

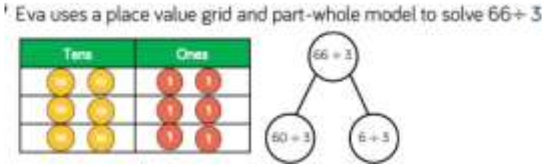
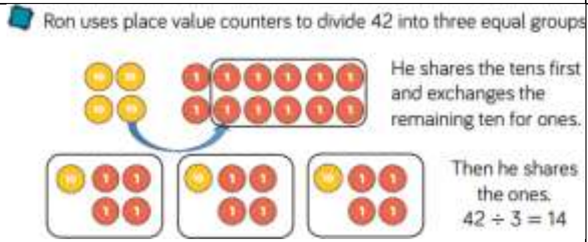
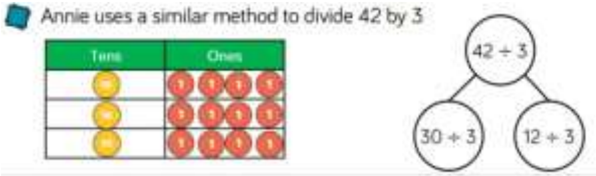
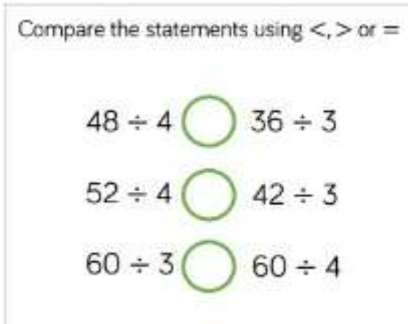
Division-
Year 1

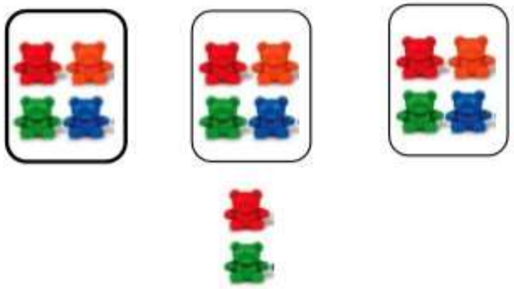
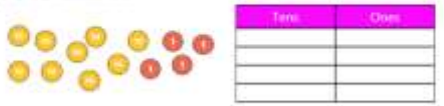
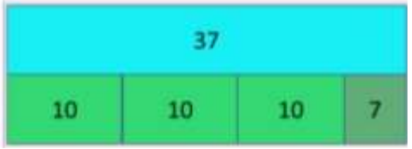
Objective and Strategy	Concret e	Pictori al	Abstra ct
Division as sharing (sharing objects into groups)	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ Children use bar modelling to show and support understanding.  $12 \div 4 = 3$	**Share 9 buns between three people.** **$9 \div 3 = 3$**
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array		

Division- Year 2			
Objective and Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.    	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	**$28 \div 7 = 4$** **Divide 28 into 7 groups. How many are in each group?**
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over		

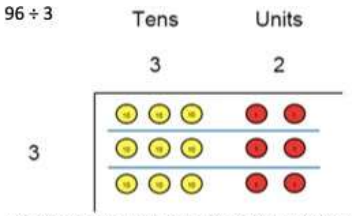
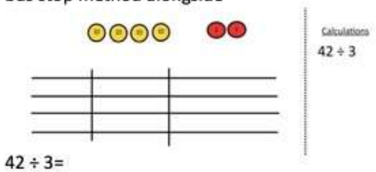
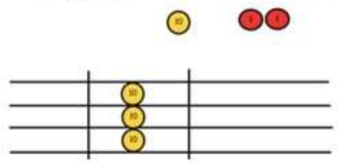
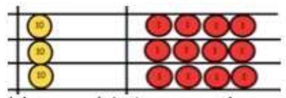
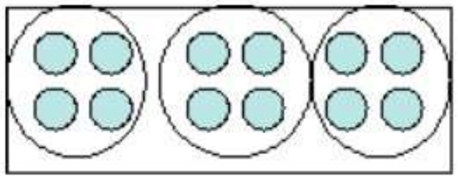
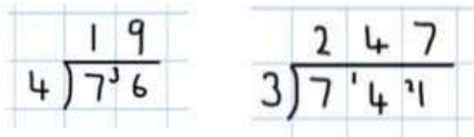
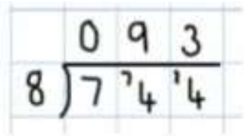
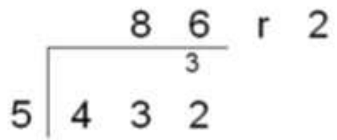
Division- Year 3

Objective and Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

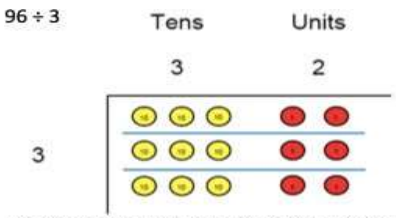
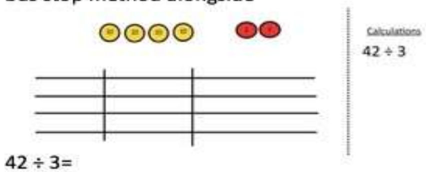
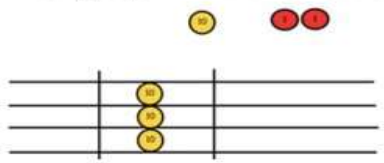
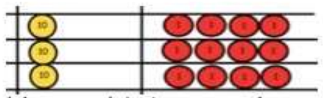
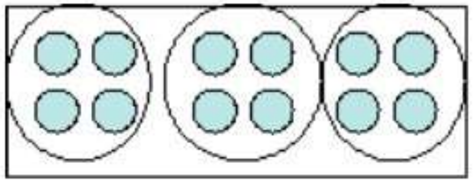
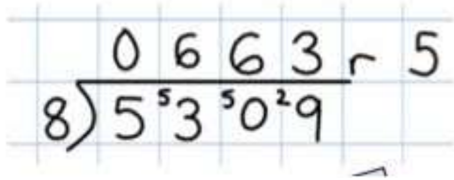
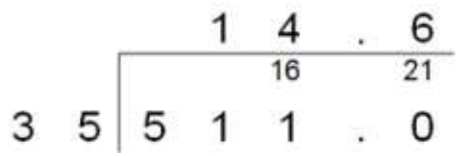
Divide 2-digit numbers by a 1-digit number by partitioning into tens and ones using a pv grid		See part- whole model	
Divide numbers that involve exchanging between the tens and ones. The answers do not have remainders.		 Children may use pictorial representation for the pv counters, alongside the part-whole model Children use their times-tables to partition the number into multiples of the divisor.	96 ÷ 8 96 ÷ 4 96 ÷ 3 96 ÷ 6 

Division with a remainder	**14 ÷ 3 =** Divide objects between groups and see how much is left over  Moving on to: Use place value counters to work out 94 ÷ 4 Did you need to exchange any tens for ones? Is there a remainder?  	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product		

Division- Year 4

Objective and Strategy	Concrete	Pictorial	Abstract
Divide up to 3 digit numbers by 1 digit. Short Division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder  Children should be aware that a 0 is used to keep place value, if the number is not divisible.  Move onto divisions with a remainder. 
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive		

Division- Year 5

Objective and Strategy	Concrete	Pictorial	Abstract
Divide at least 4 digit numbers by 1 digit. Interpret remainders appropriately for the context Short Division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	 Finally move into decimal places to divide the total accurately. 
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.		

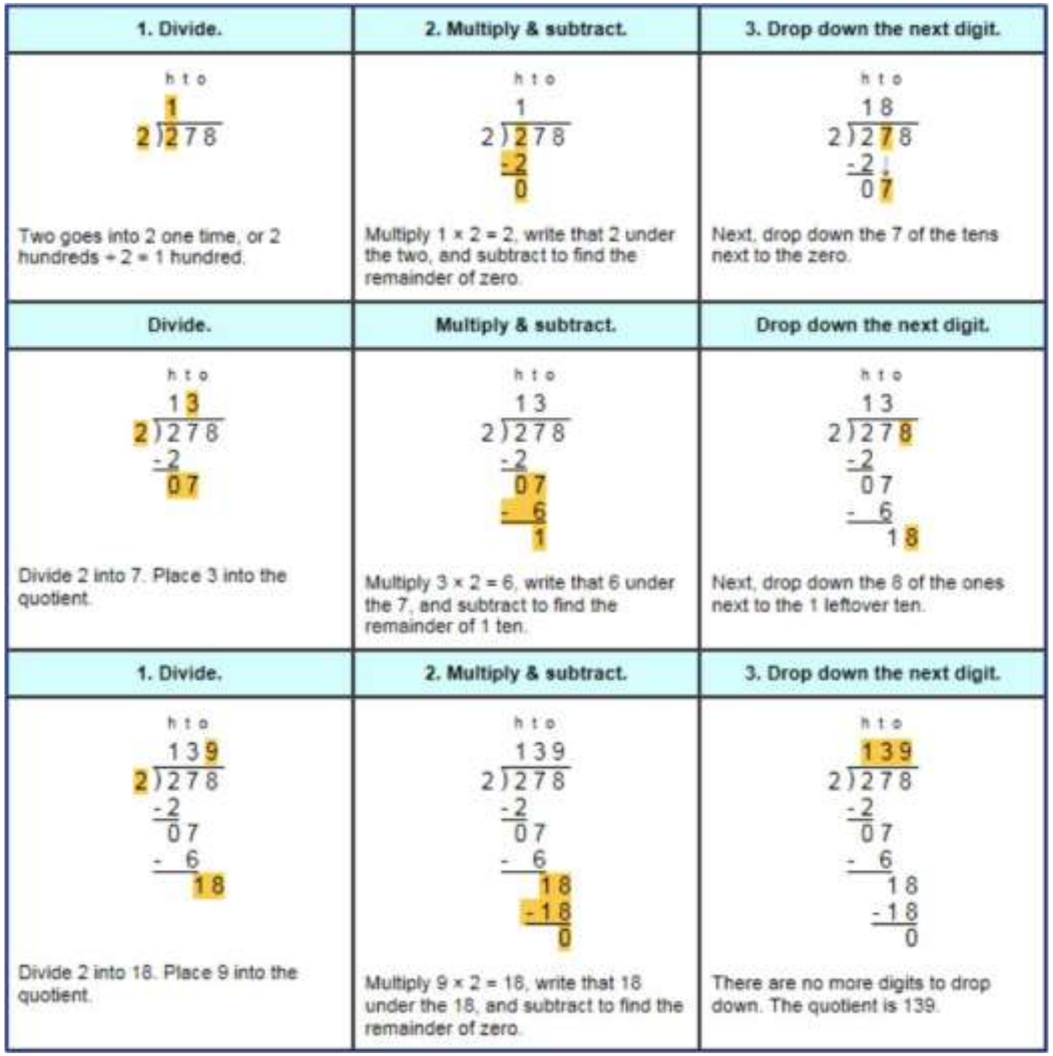
Division-Year 6

Objective and Strategy	Abstract
Long Division	Step 1 – a remainder in the ones h t o 0 4 1 R1 4) 1 6 5 4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160). 4 goes into 16 four times. 4 goes into 5 once, leaving a remainder of 1. th h t o 0 4 0 0 R7 8) 3 2 0 7 8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200). 8 goes into 32 four times (3,200 ÷ 8 = 400) 8 goes into 0 zero times (tens). 8 goes into 7 zero times, and leaves a remainder of 7. h t o 0 6 1 4) 2 4 7 − 4 3 When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3. Check: 4 × 61 + 3 = 247 th h t o 0 4 0 2 4) 1 6 0 9 − 8 1 When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1. Check: 4 × 402 + 1 = 1,609 Step 2 – a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2) 5 8 Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	t o 2 2) 5 8 - 4 1 To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	t o 2 9 2) 5 8 - 4 ↓ 1 8 Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2) 5 8 - 4 1 8 Divide 2 into 18. Place 9 into the quotient.	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0 The division is over since there are no more digits in the dividend. The quotient is 29.

Step 3 – a remainder in any of the place values

	
Vocabulary	share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)278 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)278 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 18 2)278 -2 07 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 13 2)278 -2 07 Divide 2 into 7. Place 3 into the quotient.	h t o 13 2)278 -2 07 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 13 2)278 -2 07 - 6 18 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 139 2)278 -2 07 - 6 18 Divide 2 into 18. Place 9 into the quotient.	h t o 139 2)278 -2 07 - 6 18 -18 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 139 2)278 -2 07 - 6 18 -18 0 There are no more digits to drop down. The quotient is 139.

Minimal Resources required to support the CPA approach (depending on year group):

- 10 frames (including egg boxes)
- Straws/pipe cleaners
- Bead strings (to 20 and 100)
- Rekenrek frames
- Base 10/Dienes (including magnetic to model on flip chart)
- Place value grids
- Double-sided counters
- Part-part whole templates
- Place value counters (KS2)
- Multi-link cubes